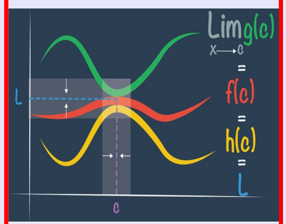
Math 261

Fall 2022

Lecture 25



$y = \sqrt{2x+1}$

$x = x(t)$

$y = y(t)$

If $\frac{dx}{dt} = 3$ find $\frac{dy}{dt}$ when $x = 4$.

$y = (2x+1)^{1/2}$ $\quad \frac{dy}{dt} = \frac{1}{\cancel{2}}(2x+1)^{-1/2} \cdot \cancel{2} \cdot \frac{dx}{dt}$

$y^2 = 2x+1$

$\cancel{2}y \cdot \frac{dy}{dt} = \cancel{2}\frac{dx}{dt}$

$y\frac{dy}{dt} = \frac{dx}{dt}$

$3\frac{dy}{dt} = 3$

$\boxed{\frac{dy}{dt} = 1}$

$\frac{dy}{dt} = \frac{1}{\sqrt{2x+1}} \cdot \frac{dx}{dt}$

$\left.\frac{dy}{dt}\right|_{x=4} = \frac{1}{\sqrt{9}} \cdot 3 = \boxed{1}$

$y = \sqrt{2x+1}$ $\quad x = 4$ $\quad y = 3$

Given $\boxed{4x^2 + 9y^2 = 36}$

$x = x(t)$ Find $\frac{dx}{dt}$ at $\left(2, \frac{2\sqrt{5}}{3}\right)$ if $\frac{dy}{dt} = \frac{1}{3}$

$y = y(t)$

$$4 \cdot 2x \cdot \frac{dx}{dt} + 9 \cdot 2y \frac{dy}{dt} = 0$$

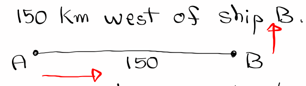
Divide by 2

$$4x \frac{dx}{dt} + 9y \frac{dy}{dt} = 0$$

$$4(2) \frac{dx}{dt} + \cancel{9}\left(\frac{2\sqrt{5}}{\cancel{3}}\right) \cdot \frac{1}{\cancel{3}} = 0$$

$$8 \frac{dx}{dt} = -2\sqrt{5} \qquad \boxed{\frac{dx}{dt} = \frac{-\sqrt{5}}{4}}$$

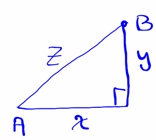
Ship A is 150 Km west of ship B.



Ship A is sailing east at 35 Km/hr.

Ship B is 〃 north at 25 Km/hr.

How fast is the distance between them changing after 4 hrs?



$\frac{dy}{dt} = 25$ Km/hr

$\frac{dx}{dt} = -35$ Km/hr

$$x^2 + y^2 = Z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2Z \frac{dZ}{dt}$$

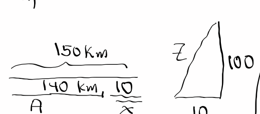
$$x \frac{dx}{dt} + y \frac{dy}{dt} = Z \frac{dZ}{dt}$$

in 4 hrs

Ship B → $4(25) = 100$ Km.

ship A → $4(35) = 140$ km

$$10(-35) + 100(25) = \sqrt{10100} \frac{dZ}{dt}$$



$$Z^2 = 100^2 + 10^2 = 10\,000 + 100$$

$$Z^2 = 10\,100$$

$$Z = \sqrt{10\,100}$$

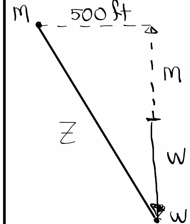
$$\frac{dZ}{dt} = \frac{10(-35) + 100(25)}{\sqrt{10\,100}}$$

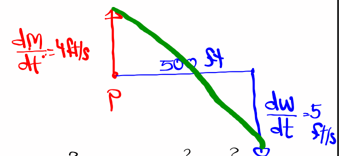
$\boxed{\frac{dZ}{dt} = \quad \text{Km/hr}}$

A man starts walking north @ 4ft/s from a point P.

5 minutes later, A woman starts walking South from a point 500 ft east of P at the rate of 5 ft/s.

At what rate are these two moving apart 15 minutes after the woman starts walking?





$\frac{dm}{dt} = 4\text{ ft/s}$

$\frac{dw}{dt} = 5\text{ ft/s}$

$500^2 + (m+w)^2 = Z^2$

$0 + \cancel{2}(m+w)\cdot\left[\frac{dm}{dt} + \frac{dw}{dt}\right] = \cancel{2}Z\frac{dZ}{dt}$

$(4800+4500)\cdot[4+5] = \quad \frac{dZ}{dt}$

find m

$(20)\cdot 4(60) = 4800$

$5 + 15$

find w

$15(5)(60) = 4500$

Solve for $\frac{dZ}{dt} = \boxed{}$

find Z

$500^2 + (m+w)^2 = Z^2$

$500^2 + (4800+4500)^2 = Z^2 \rightarrow \boxed{Z = }$

A particle is moving along the curve given by $y = 2\sin\frac{\pi x}{2}$

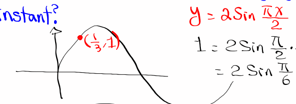
At the point $(\frac{1}{3}, 1)$, $\frac{dx}{dt} = \sqrt{10}$ cm/s.

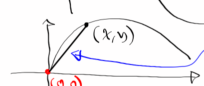
How fast is the distance between the Particle and the origin changing at that instant?

$y = 2\sin\frac{\pi x}{2}$

$1 = 2\sin\frac{\pi}{2}\cdot\frac{1}{3}$

$= 2\sin\frac{\pi}{6} = 2\cdot\frac{1}{2} = 1$ ✓





$Z = \sqrt{(x-0)^2 + (y-0)^2}$

$Z^2 = x^2 + y^2$

we want $\frac{dZ}{dt}$ at $(\frac{1}{3}, 1)$

$Z^2 = x^2 + \left(2\sin\frac{\pi x}{2}\right)^2$

$Z^2 = x^2 + 4\sin^2\frac{\pi x}{2}$

$Z^2 = (\frac{1}{3})^2 + 4\sin^2\frac{\pi}{2}\cdot\frac{1}{3}$

$= \frac{1}{9} + 4\sin^2\frac{\pi}{6}$

$= \frac{1}{9} + 4\cdot\frac{1}{4} \rightarrow Z^2 = \frac{10}{9}$

$Z = \frac{\sqrt{10}}{3}$

$\frac{dZ}{dt} = ?$

$\cancel{2}Z\frac{dZ}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{4}^{2}\cdot\cancel{2}\cdot\sin\frac{\pi x}{2}\cdot\cos\frac{\pi x}{2}\cdot\frac{\pi}{\cancel{2}}\cdot\frac{dx}{dt}$

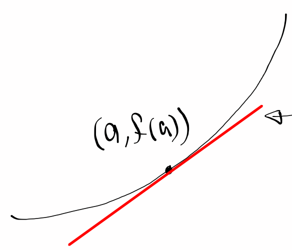
$Z\frac{dZ}{dt} = x\frac{dx}{dt} + \underbrace{2\pi\cdot\sin\frac{\pi x}{2}\cdot\cos\frac{\pi x}{2}}_{\pi\sin 2(\frac{\pi x}{2})}\frac{dx}{dt}$

$Z\frac{dZ}{dt} = x\frac{dx}{dt} + \pi\sin\pi x\frac{dx}{dt}$

$\boxed{\frac{\sqrt{10}}{3}}\frac{dZ}{dt} = \frac{1}{3}\cdot\sqrt{10} + \pi\sin\frac{\pi}{3}\cdot\sqrt{10}$

$\frac{\sqrt{10}}{3}\frac{dZ}{dt} = \frac{\sqrt{10}}{3} + \frac{\pi\sqrt{10}\cdot\sqrt{3}}{2}$

$\frac{dZ}{dt} = \boxed{}$



$$y - y_1 = m(x - x_1)$$
$$y - f(a) = f'(a)(x - a)$$
$$y = f(a) + f'(a)(x - a)$$

Near the tangent Point $\quad f(x) \approx f(a) + f'(a)(x - a)$

Estimate $\sqrt{4.1}$ $\quad$ Consider $f(x) = \sqrt{x}$ & $a = 4$

using Calc.

$\sqrt{4.1} \approx 2.02$

$f(4) = 2$

$f'(x) = \dfrac{1}{2\sqrt{x}}$

$f'(4) = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

$$\sqrt{x} \approx f(4) + f'(4)(x - 4)$$
$$\sqrt{x} \approx 2 + \frac{1}{4}(x - 4)$$
$$\sqrt{4.1} \approx 2 + \frac{1}{4}(4.1 - 4)$$
$$\approx 2 + .25(.1)$$
$$\approx 2 + .025 = \boxed{2.025}$$